Growth Curve Modeling

Day 2
Applications of Growth Curve Models
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Independent Variables

- Think about what unit at which you hypothesize the predictor acts
  - Does this predictor characterize the person? If yes, it is a **time-invariant** predictor
  - Does this predictor characterize a point of time in the person's life? If yes, it is a **time-varying** predictor

Time-Invariant Predictors

Do we think that the trait

- leads some group to have higher values at the outset than another group, but both groups generally change at a similar rate?
- leads groups to change at different rates after having no meaningful difference in where they started?
- is associated with *both* different starting values and different rates of change?

Time Invariant Predictors

Basic growth trajectory model (random intercepts & slopes):

$$y_{ti} = \beta_{0i} + \beta_{1i}(\text{month})_t + e_{ti}$$
$$\beta_{0i} = \gamma_{00} + r_{0i}$$
$$\beta_{1i} = \gamma_{10} + r_{1i}$$

Growth trajectory model with time-invariant predictors:

$$y_{ti} = \beta_{0i} + \beta_{1i}(\text{month})_t + e_{ti}$$
$$\beta_{0i} = \gamma_{00} + \gamma_{01}W_i + r_{0i}$$
$$\beta_{1i} = \gamma_{10} + \gamma_{11}W_i + r_{1i}$$

We are just making a regression out of the betas!!!

Time Invariant Predictors

Growth trajectory model with time-invariant predictors:

$$y_{ti} = \beta_{0i} + \beta_{1i}(\text{month})_t + e_{ti}$$
$$\beta_{0i} = \gamma_{00} + \gamma_{01}W_i + r_{0i}$$
$$\beta_{1i} = \gamma_{10} + \gamma_{11}W_i + r_{1i}$$

Reduced-form model:

$$\underbrace{y_{ti}}_{\text{outcome}} = \underbrace{\gamma_{00} + \gamma_{01}W_{1i} + (\gamma_{10} + \gamma_{11}W_{1i})\,(\text{month})_t}_{\text{deterministic}} + \underbrace{r_{0i} + r_{1i}(\text{month})_t + e_{ti}}_{\text{stochastic}}$$

Effect on the Intercept

If we think that the trait leads some group to have higher values at the outset than another group, but both groups generally change at a similar rate:

$$\underbrace{y_{ti}}_{\text{outcome}} = \underbrace{\gamma_{00} + \gamma_{01} W_{1i} + (\gamma_{10} + \cancel{\gamma_{11} W_{1i}})\,(\text{month})_t}_{\text{deterministic}} + \underbrace{r_{0i} + r_{1i}(\text{month})_t + e_{ti}}_{\text{stochastic}}$$

Effect on the Slope

If we think that the trait leads groups to change at different rates after having no meaningful difference in where they started

$$\underbrace{y_{ti}}_{\text{outcome}} = \underbrace{\gamma_{00} + \cancel{\gamma_{01} W_{1i}} + (\gamma_{10} + \gamma_{11} W_{1i})\,(\text{month})_t}_{\text{deterministic}} + \underbrace{r_{0i} + r_{1i}(\text{month})_t + e_{ti}}_{\text{stochastic}}$$

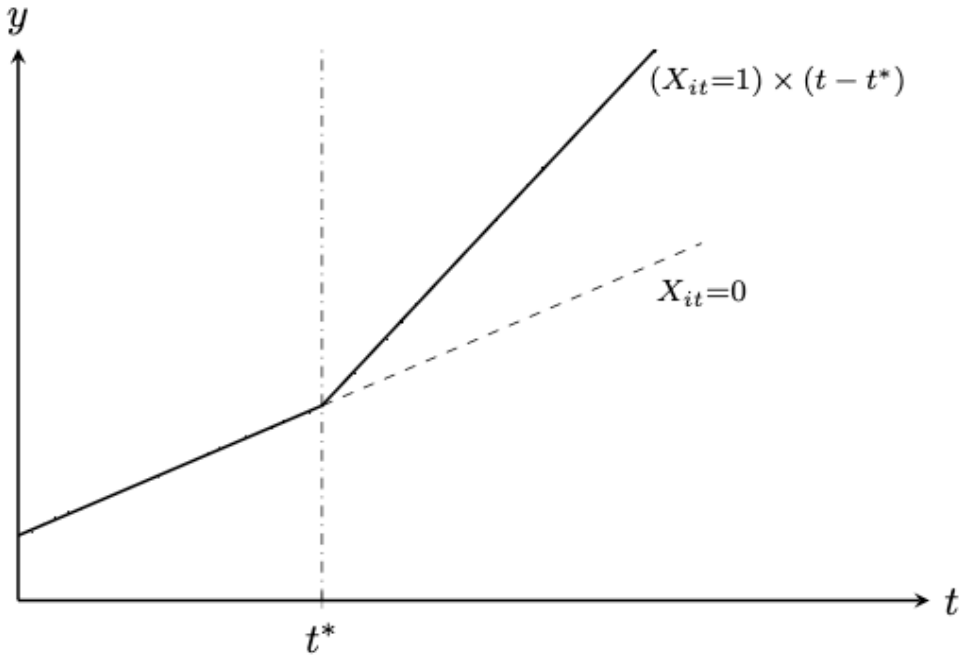
Effect on Intercepts and Slopes

If we think that the trait is associated with *both* different starting values and different rates of change?

$$\underbrace{y_{ti}}_{\text{outcome}} = \underbrace{\gamma_{00} + \gamma_{01} W_{1i} + (\gamma_{10} + \gamma_{11} W_{1i})\,(\text{month})_t}_{\text{deterministic}} + \underbrace{r_{0i} + r_{1i}(\text{month})_t + e_{ti}}_{\text{stochastic}}$$

All growth curve models are regressions!!!

Altered Rate of Change



Intervention (Interruption) Altering Rate of Improvement

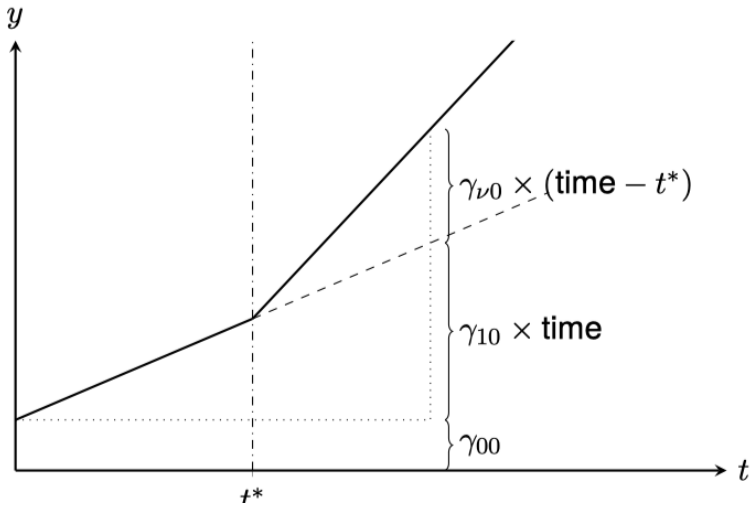
$$\underbrace{y_{ti}}_{\text{outcome}} = \underbrace{\gamma_{00} + \gamma_{\nu 0}\nu_{ti} + \gamma_{10}(\text{time})_{ti}}_{\text{deterministic}} + \underbrace{\rho_{0i} + \rho_{\nu 0}\nu + \rho_{1i}(\text{time}_{ti}) + \epsilon_{ti}}_{\text{stochastic}}$$

where:

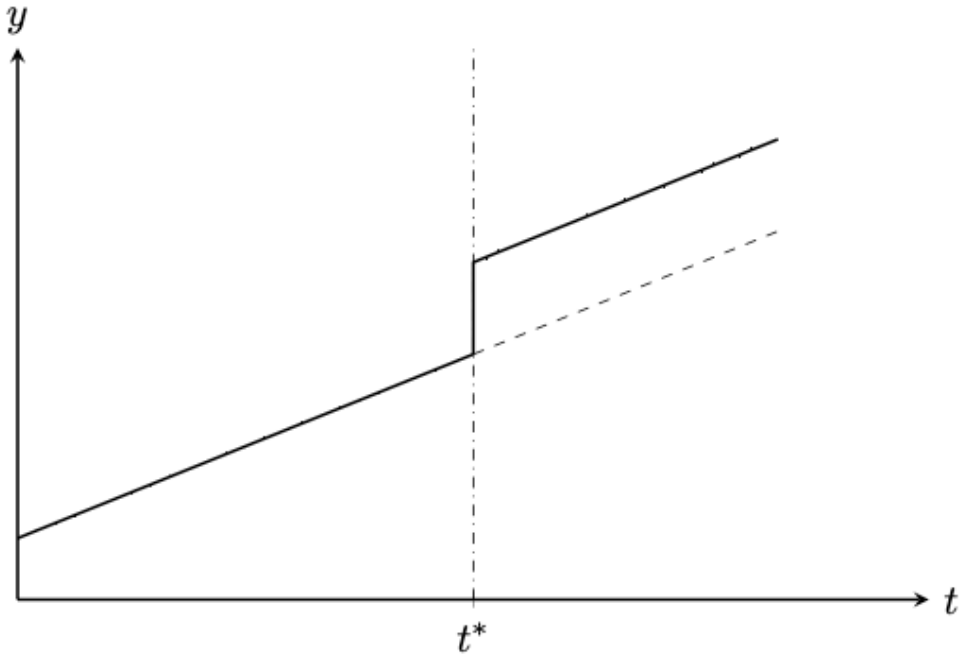
$$\nu_{ti} = \begin{cases} \text{time}_{ti} - \text{time}_i^* & \text{if intervention for person i happened at or before time t} \\ 0 & \text{if intervention has not yet happened} \end{cases}$$

Altered Rate of Change

$$\underbrace{y_{ti}}_{\text{outcome}} = \underbrace{\gamma_{00} + \gamma_{\nu 0}\nu_{ti} + \gamma_{10}(\text{time})_{ti}}_{\text{deterministic}} + \underbrace{\rho_{0i} + \rho_{\nu 0}\nu + \rho_{1i}(\text{time}_{ti}) + \epsilon_{ti}}_{\text{stochastic}}$$



Deviation in Level



Deviation in Level

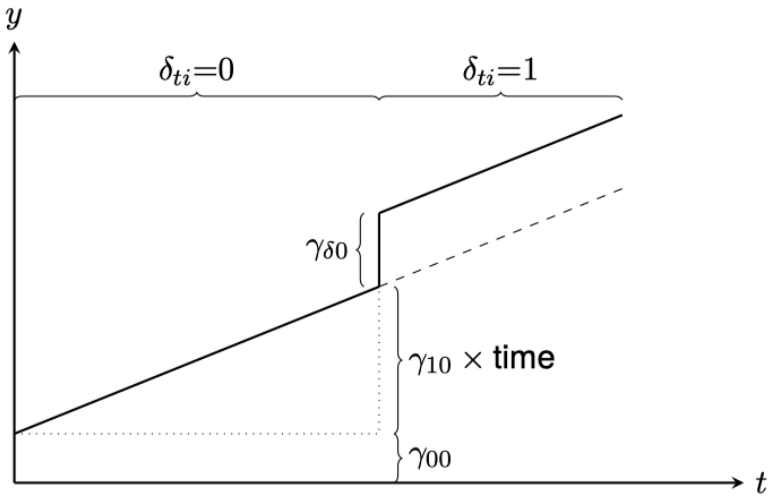
$$\underbrace{y_{ti}}_{\text{outcome}} = \underbrace{\gamma_{00} + \gamma_{\delta 0}\delta_{ti} + \gamma_{10}(\text{time})_{ti}}_{\text{deterministic}} + \underbrace{\rho_{0i} + \rho_{\delta i}\delta + \rho_{1i}(\text{time}_{ti}) + \epsilon_{ti}}_{\text{stochastic}}$$

where:

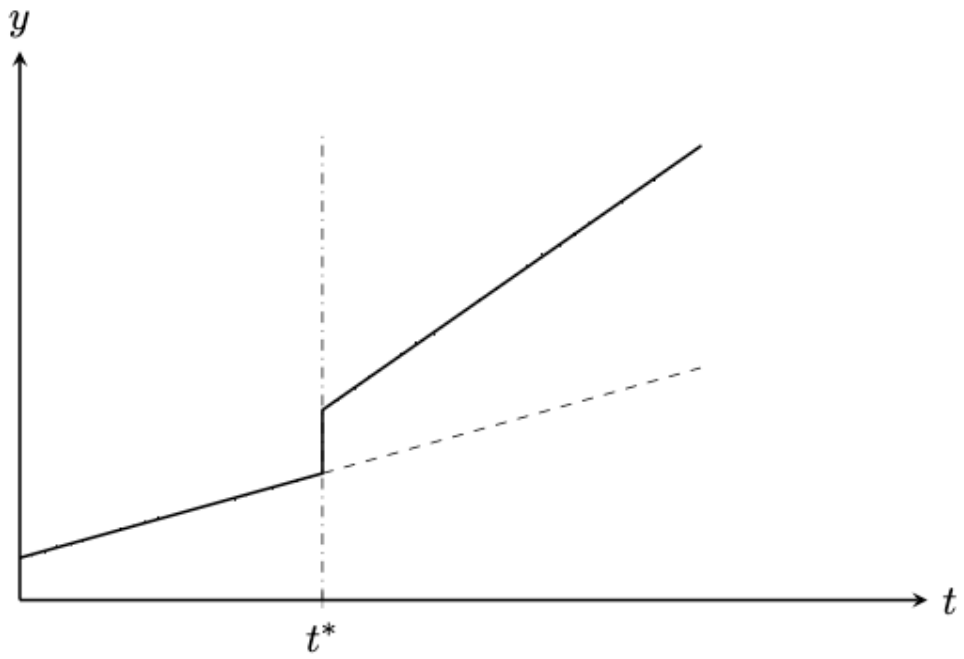
$$\delta_{ti} = \begin{cases} 1 & \text{if intervention for person i happened at or before time t} \\ 0 & \text{otherwise} \end{cases}$$

Intervention with Immediate Change in Level

$$\underbrace{y_{ti}}_{\text{outcome}} = \underbrace{\gamma_{00} + \gamma_{\delta 0}\delta_{ti} + \gamma_{10}(\text{time})_{ti}}_{\text{deterministic}} + \underbrace{\rho_{0i} + \rho_{\delta i}\delta + \rho_{1i}(\text{time}_{ti}) + \epsilon_{ti}}_{\text{stochastic}}$$

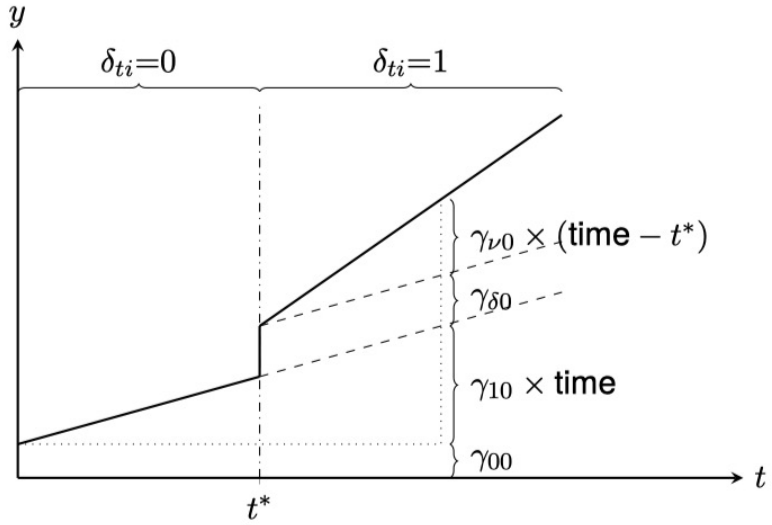


Both Altered Rate of Change and Deviation in Level



Both Altered Rate of Change and Deviation in Level

$$\underbrace{y_{ti}}_{\text{outcome}} = \underbrace{\gamma_{00} + \gamma_{\delta 0}\delta_{ti} + \gamma_{\nu 0}\nu_{ti} + \gamma_{10}(\text{time})_{ti}}_{\text{deterministic}} + \underbrace{\rho_{0i} + \rho_{\delta i}\delta_{ti} + \rho_{\nu i}\nu_{ti} + \rho_{1i}(\text{time}_{ti}) + \epsilon_{ti}}_{\text{stochastic}}$$



Review

Base Models

- All models have three components, ensure you have thought about how all three work:
  1. Outcome (datum-level variation)
  2. Deterministic (data-level variation)
  3. Stochastic (datum-level variation)
- Our goal is to minimize errors at different levels ("fancy means")
- Think about how your data vary over units and over time
- Estimating models of your data
  - Simulate your model and make guesses about coefficients
  - Analyze collected data to using simulated model & code

Differences in Variation

- Consider *interpretation* of analysis when choosing intercept
- Think carefully about how time should be measured
- Describe data by examining trends and deciding on shape of change
- Decide on covariates to include in the models with careful attention to how you think that they affect the outcome
- Determine whether data follow a single trajectory with random variation or follow categorically distinct trajectories

Thank You!

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